MATH 464 (THEORY OF PROBABILITY)
HOMEWORK 7

FALL 2017

Due on: Thursday 10-19-2017.

(1) Show that if X and Y take only two values 0 and 1 and $\mathbb{E}(XY) = \mathbb{E}(X)\mathbb{E}(Y)$, then X and Y are independent.

(2) Is $M(t) = \frac{e^t+e^{-t}}{6} + \frac{2}{3}$ a moment generating function of a random variable? If yes, find the corresponding probability mass function.

(3) Suppose that $X_1, X_2, \ldots, X_n > 0$ are independent identically distributed random variables. Let $m < n$. Find
$$\mathbb{E}\left(\frac{X_1 + \ldots + X_m}{X_1 + \ldots + X_n}\right).$$

(4) Show that $p(n) = \frac{1}{n(n+1)}$ for $n = 1, 2, \ldots$ is a probability mass function for a discrete random variable X, find $\mathbb{E}(X)$.

(5) Let F_1 and F_2 be distribution functions of some random variables, show that for every $0 \leq \alpha \leq 1$, the function
$$F = \alpha F_1 + (1 - \alpha)F_2$$
is a distribution function of some random variable.